Published by Institute of Physics Publishing for SISSA

Received: *May 5, 2005*
Revised: *June 15, 2005*
Accepted: *June 21, 2005*
Published: *July 20, 2005*

Kähler corrections and softly broken family symmetries

Steve F.King and Iain N.R. Peddie

School of Physics and Astronomy, U. of Southampton
Southampton SO17 1BJ, U.K.
E-mail: sfk@hep.phys.soton.ac.uk, i.peddie@hep.phys.soton.ac.uk

Graham G. Ross

The Rudolf Peierls Centre for Theoretical Physics
1 Keble Road, Oxford, OX1 3NP, U.K.
E-mail: g.ross1@physics.ox.ac.uk

Liliana Velasco-Sevilla

Michigan Center for Theoretical Physics, Randall Laboratory, U. of Michigan
500 E University Ave., Ann Arbor, MI 48109, U.S.A.
E-mail: lvelsev@umich.edu

Oscar Vives

Theory Division, CERN
CH-1211, Geneva 23, Switzerland
E-mail: oscar.vives@cern.ch

Abstract: Spontaneously broken family symmetry provides a promising origin for the observed quark and lepton mass and mixing angle structure. In a supersymmetric theory such structure comes from a combination of the contributions from the superpotential and the Kähler potential. The superpotential effects have been widely studied but relatively little attention has been given to the effects of the Kähler sector. In this paper we develop techniques to simplify the analysis of such Kähler effects. Using them we show that in the class of theories with an hierarchical structure for the Yukawa couplings the Kähler corrections to both the masses and mixing angles are subdominant. This is true even in cases that texture zeros are filled in by the terms coming from the Kähler potential.

Keywords: Supersymmetry Phenomenology, Supergravity Models, Quark Masses and SM Parameters.

JHEP07(2005)049

Contents

1. Introduction **1**

2. The Kähler metric **2**

3. The Kähler corrections to Yukawa couplings **4**
- 3.1 The form of the Yukawa coupling matrix 4
- 3.2 The Kähler corrections 5
  - 3.2.1 No additional texture zeros 7
  - 3.2.2 Additional texture zeros 8

4. Kähler corrections to the mass matrix eigenvalues and mixing angles **9**
- 4.1 Quark and charged lepton masses and mixing angles. 9
- 4.2 Neutrino masses and mixing angles 11
- 4.3 Soft SUSY breaking masses and mixing angles 12

5. Conclusions **13**

1. Introduction

The origin of the structure of the fermion Yukawa couplings is one of the most intriguing puzzles left unanswered by the Standard Model. The hierarchical pattern of fermion masses and quark mixing angles strongly suggests the existence of a spontaneously broken family symmetry with the order parameter of breaking (the vacuum expectation value (vev) of one or more scalar familon fields) providing the small expansion parameter(s). This has been the most popular strategy to try to improve our understanding of the flavour structures in nature. In this scheme the (usually Supersymmetric) Standard Model is extended by a gauge or global family symmetry G_F which is then spontaneously broken. The Yukawa couplings (or just those associated with the two lighter generations) are not allowed in the limit of unbroken family symmetry but are filled in by higher dimension operators involving powers of the familon field(s). Thus below the scale of G_F breaking, we have an effective theory resembling the Supersymmetric Standard Model where the Yukawa couplings (with the possible exception of the third family) and all the different flavour structures are given by non-renormalisable operators in the superpotential of the kind,

$$\psi\psi^c H \left(\frac{\langle\theta\rangle}{M}\right)^n , \tag{1.1}$$

where ψ and ψ^c denote quark/lepton superfields, H is a Higgs superfield, M is the heavy messenger mass corresponding to the intermediate state in the Froggatt-Nielsen mechanism and $\langle\theta\rangle$ is the familon vev that breaks G_F such that $\langle\theta\rangle/M$ is a small expansion parameter [1]. In the following, we do not specify the family group or the exact mechanism of symmetry breaking as our conclusions are equally applicable to (abelian or non-abelian) flavour theories generating a given structure in the Yukawa couplings [2].

The generation of Yukawa couplings (or other holomorphic couplings in the superpotential) though non-renormalisable operators is not the only effect of integrating out the heavy fields in the low energy effective theory. It is well known that the non-holomorphic couplings involving the kinetic terms and gauge couplings also receive corrections from the flavour breaking terms. This implies a non canonical Kähler potential, different from the identity in flavour space. In determining the physical implications of the theory it is much simpler to work in a theory with canonical kinetic terms and this can be done by choosing a linear combination of the chiral superfields such that the new fields have a canonical Kähler potential [3]. As originally shown by Leurer, Nir and Seiberg [4] this transformation in the chiral superfields consists of a rotation in flavour space and a rescaling of the fields. However, even after this field redefinition, we can still perform further arbitrary unitary rotations of the chiral superfields which will preserve the canonical form of the Kähler potential. Clearly any superfield field redefinitions in the Kähler potential must be performed consistently for all the superfields in the theory and this will result in a transformation of the superpotential couplings when written in terms of the new chiral superfields. This transformation of the Yukawa couplings is the main subject of this work and we are especially interested in the observable effects of this transformation on the physical masses and mixing angles. In fact, in the literature it is often stated that these field redefinitions can have very important observable effects in quark and squark mixings [5–7]. That this is not the case in specific models has been stressed in [4, 8]. Here we generalise this result and show that, at least for the case of an hierarchical Yukawa textures for the up and down sectors, the effect of the Kähler potential, is always sub-dominant and cannot change the structure coming from the superpotential. In the presence of a hierarchical texture ordered by an underlying family symmetry, the structure of the Kähler potential is such that the off-diagonal elements are given by powers of the same small expansion parameter that generates the hierarchy in the Yukawa matrices. Under these conditions we show that we can choose an upper triangular form for the inverse of the square root of the Kähler metric which brings the fields to the canonical basis. Using this form we can prove the subdominance of the Kähler corrections to the Yukawa matrices. Even in cases without a clear hierarchy we show that only unknown coefficients $\mathcal{O}(1)$ can be changed without modifying the structure of the observable mixings and masses, consistent with the results of [4]. Therefore our conclusions apply to all flavour models with hierarchical Yukawa textures considered in the literature.

2. The Kähler metric

After the flavour symmetry is spontaneously broken we obtain a certain Yukawa texture given by non-renormalisable operators which are functions of the flavon vevs as in eq. (1.1). In the same way the effective Kähler potential will be a general non-renormalisable real function invariant under all the symmetries of the theory coupling the superfield combinations $\psi_i^\dagger \psi_j$ to the flavon fields, and similarly for ${\psi^c}_i^\dagger \psi_j^c$, where i,j are flavour indices. The terms $\psi_i^\dagger \psi_i$, ${\psi^c}_i^\dagger \psi_i^c$ without flavon superfields are clearly invariant under gauge, flavour and global symmetries and hence give rise to a family universal contribution. However, family symmetry breaking terms involving flavon superfields give rise to important corrections [9, 10, 4]. In fact, it is interesting to notice that, due to the non-holomorphicity of the Kähler potential, new terms are allowed with different structure from the terms that appear in the Yukawa couplings of the superpotential.

In general the matter fields do not have canonical wave functions (kinetic terms) in the symmetry eigenstate basis $\hat{\psi}_i^c$, $\hat{\psi}_j$ [7]. Rather, flavon field vevs contribute to the diagonal terms and also generate new flavour off-diagonal entries. Thus, we have now non-canonical kinetic terms and we must redefine the fields to obtain canonical kinetic terms. The effect of these redefinitions, which can be regarded as wave function corrections, on the Yukawa couplings and other couplings in the theory may be determined after this field redefinition, $\hat{\psi} = N\psi$.

To obtain canonical kinetic terms we have to redefine the fields to go to the canonical basis by the inverse of the square root of the Kähler metric K given by

$$\hat{\psi}^\dagger K \hat{\psi} = (N\psi)^\dagger (N^{-1})^\dagger N^{-1} N\psi \,. \tag{2.1}$$

Thus $K = (N^{-1})^\dagger N^{-1}$ and hence $N = K^{-1/2}$, as claimed above. Using Supergravity (SUGRA) equations, the Kähler metric is obtained as $K_{\bar{a}b} = \partial^2 G/(\partial \Phi_a^\dagger \partial \Phi^b)$ with G the Kähler function and it determines both the Kinetic terms and the non-canonically normalised soft scalar mass squared matrices $\hat{m}^2_{\bar{a}b}$. In SUGRA, where $K_{\bar{a}b}$ represents a metric, N^{-1} is also a hermitean matrix, such that $N^{-1} = (N^{-1})^\dagger$ and hence it can be conventionally written as [6, 7]

$$K = (N^{-1})^\dagger N^{-1} = V^\dagger X^2 V \Leftrightarrow N^{-1} = V^\dagger X V$$

with V a unitary matrix diagonalising the hermitean matrix K and X the square root of the eigenvalues of K. We call this solution the "standard" form of N^{-1}. Note that if N^{-1} is a solution of eq. (2.1) then also $R.N^{-1}$ is a solution of eq. (2.1), with R a unitary matrix. Of course physical quantities will not depend on R and for any choice we must always obtain the same physical result. This is due to the invariance of the lagrangian under the so-called Weak Basis Transformations (WBT) [11, 12]. The theory is invariant if we transform the fields as,

$$q_L = R_q q_L' \quad ; \quad u_R = R_u u_R' \,; \qquad d_R = R_d d_R'$$

where R_q, R_u and R_d are transformations from the global unitary groups $\mathrm{U}(3)_L$, $\mathrm{U}(3)_{u_R}$ and $\mathrm{U}(3)_{d_R}$ respectively, while simultaneously the Yukawa couplings are transformed as,

$$Y_u' = R_q^\dagger Y_u R_u \qquad Y_d' = R_q^\dagger Y_d R_d \,. \tag{2.2}$$

Therefore when we choose the different R_a all we are doing is to choose a particular weak basis where we write our theory and the physical results are absolutely independent of this choice. However, it is very useful to choose the unitary transformation R in the definition of $N = K^{-1/2}$ to get a simpler form for this transformation. The form that proves to be useful is the Cholesky decomposition of an hermitean matrix. It is always possible to write an hermitean matrix as $K = U^\dagger U$ in terms of an upper U triangular matrix,

$$K = \begin{pmatrix} K_{11} & K_{12} & K_{13} \\ K_{12}^* & K_{22} & K_{23} \\ K_{13}^* & K_{23}^* & K_{33} \end{pmatrix} = U^\dagger U = \begin{pmatrix} u_{11} & 0 & 0 \\ u_{12}^* & u_{22} & 0 \\ u_{13}^* & u_{23}^* & u_{33} \end{pmatrix} \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix} \tag{2.3}$$

This equation is very easy to solve,

$$\begin{aligned} &u_{11} = \sqrt{K_{11}} \quad u_{12} = \frac{K_{12}}{\sqrt{K_{11}}} \quad u_{13} = \frac{K_{13}}{\sqrt{K_{11}}} \\ &u_{22} = \sqrt{K_{22} - \frac{|K_{12}|^2}{K_{11}}} \quad u_{23} = \frac{K_{23}K_{11} - K_{13}K_{12}^*}{\sqrt{K_{22}K_{11}^2 - K_{11}|K_{12}|^2}} \quad u_{33} = \sqrt{K_{33} - |u_{23}|^2 - |u_{13}|^2}\,. \end{aligned} \tag{2.4}$$

The inverse of this upper triangular matrix is also upper triangular, and it is also easily obtained. Obviously we could have chosen to use lower triangular matrices L instead of the upper triangular matrices U and the explicit form of the L would then have been obtained in a similar way in terms of K.

This form for the square root of the Kähler matrix is different from the "standard" form used in the literature [6, 7]. Clearly the "standard" form is related to our triangular form by an unobservable WBT and therefore the two forms are physically indistinguishable. However it is evident from Eq. (2.4) that from the point of view of calculability it is much simpler to obtain the triangular form than the "standard" form.

3. The Kähler corrections to Yukawa couplings

3.1 The form of the Yukawa coupling matrix

To proceed we need to know the form of the Yukawa couplings coming from the superpotential. A fit to the data using a form for the Yukawa matrices where the smallness of CKM mixing angles is due to the smallness of the off-diagonal entries with respect to the relevant diagonal entry yields the structure [13],

$$Y_d \propto \begin{pmatrix} 0 & \bar\varepsilon^3 & \bar\varepsilon^3 \\ . & \bar\varepsilon^2 & \bar\varepsilon^2 \\ . & . & 1 \end{pmatrix}, \qquad Y_u \propto \begin{pmatrix} 0 & \varepsilon^3 & \varepsilon^3 \\ . & \varepsilon^2 & \varepsilon^2 \\ . & . & 1 \end{pmatrix}$$

with the expansion parameters $\bar\varepsilon = 0.15$ and $\varepsilon = 0.05$. Some non-abelian family symmetry models can provide such a structure quite naturally [14, 15]. Here we have suppressed coefficients of $\mathcal{O}(1)$. This structure has $Y_{kj} < Y_{ij}$ for $i > k$ and $j \geq i$ and is unique if the contribution to the left-handed mixing angles from the elements below the diagonal are negligible. If one relaxes this constraint then it is possible for some of the entries to be smaller or zero (texture zeros). We will discuss both these possibilities. To do so let us first note that, although there are no direct bounds on the Yukawa couplings below the diagonal from (right-handed) mixing angles, we can obtain some upper bounds on these entries from their contributions to the mass eigenvalues. Just requiring that the determinant of the down Yukawa matrix is $\bar\varepsilon^6 = 1 \times \frac{m_s}{m_b} \times \frac{m_d}{m_b}$ we arrive to the conclusion that $Y^d_{21} \leq \bar\varepsilon^3$, $Y^d_{31} \leq \bar\varepsilon$ and $Y^d_{32} \leq 1$, assuming no cancellation between different contributions to the determinant. With this the most general hierarchical down-quark Yukawa structure consistent with the masses and mixing angles is

$$Y_d \propto \begin{pmatrix} \leq \bar\varepsilon^4 & a\,\bar\varepsilon^3 & b\,\bar\varepsilon^3 \\ \leq \bar\varepsilon^3 & c\,\bar\varepsilon^2 & d\,\bar\varepsilon^2 \\ \leq \bar\varepsilon & \leq 1 & 1 \end{pmatrix}. \tag{3.1}$$

Not all of the four coefficients a, b, c, d must be $\mathcal{O}(1)$ allowing for the possibility of additional texture zeros. In principle the Y_u structure could also be described by this structure with the only replacement $\bar\varepsilon \to \varepsilon$. As explained below, given that the SM gauge group does not relate the up and down right handed sectors, this structure with different expansion parameters in Y^u and Y^d emerges naturally in a multitude of flavour models both with abelian and non-abelian symmetries, for example in a U(1) model with Frogatt-Nielsen messenger fields of different masses [16]. However, our results below do not require the presence of two different expansion parameters for the up and the down sector and we could reproduce the same fit with different powers of the same expansion parameter [17].[1]

In this paper we consider the case that this hierarchical structure eq. (3.1) is reproduced by the terms of the superpotential in the symmetry basis and we show that the effect of the Kähler potential is then always subdominant in its effects on the masses and mixing angles.

3.2 The Kähler corrections

It proves to be useful in most realistic models to go to the canonically normalised basis by redefining the fields by a wave function normalisation matrix chosen to have the upper triangular form, as discussed above. Using this form the correction to the Yukawa coupling matrix in the Standard Model (SM) is of the form

$$H\hat{\overline{\psi}}_L\, Y\, \hat\psi_R \;\equiv\; H\,\hat\psi^*_{L\,i}\, Y_{ij}\, \hat\psi_{R\,j} \;=\; H\,\psi^*_{L\,k} N^*_{L\,ik}\, Y_{ij}\, N_{R\,jm}\psi_{R\,m} \;=\; H\,\psi^*_{L\,k}\, Y^t_{km}\,\psi_{R,m}$$

If we consider, for the moment, only the transformation on the Left Handed (LH) fields using our triangular matrices, with $N = U$, the total (t) Yukawa is,

$$Y^t_{ij} = \sum_{i\geq k} N^*_{ki} Y_{kj} \simeq N^*_{ii} Y_{ij} + \sum_{i>k} N^*_{ki} Y_{kj}\,. \tag{3.2}$$

As may be seen in eq. (1.1) the expansion parameters are given by terms of the form $<\theta>/M$ where M is the messenger mass. In the superpotential the expansion parameters come from both the LH and Right Handed (RH) sectors. The expansion parameters, ε

[1] Strictly speaking the observed values of up-quark masses and CKM mixing angles would still allow $(Y_u)_{23} = \mathcal{O}(\varepsilon)$ and/or $(Y_u)_{13} = \mathcal{O}(\varepsilon^2)$ if simultaneously $(Y_u)_{32} = \mathcal{O}(\varepsilon)$ and $(Y_u)_{31} = \mathcal{O}(\varepsilon^2)$. Given that this structure is hierarchical, all the results presented in the following are also valid in this case.

JHEP07(2005)049

and $\bar{\varepsilon}$, for the up and down sectors[2] in the superpotential may differ as the SM gauge group does not relate the up and down right handed quark sectors. However the contribution from the LH sector to the mass matrix structure must be equal in the up and down sectors due to the $\mathrm{SU}(2)_L$ gauge symmetry. Thus its contribution cannot be larger than ε, the smaller of the two (right handed) expansion parameters. This implies that the Kähler rescaling matrix in the LH sector, N^L_{ik}, has a strong hierarchy controlled by the small parameter ε with $N^L_{ii} \simeq 1$ and $N^L_{ik} \leq \epsilon$ for the non-zero entries of the upper triangular form. Notice that the smallness of off-diagonal elements in N_{ij} (and K_{ij}) is necessary in any model where the hierarchy of the Yukawa matrices is due to the presence of an underlying family symmetry, either abelian or non-abelian. This is specially simple in abelian models where the hierarchy in the Yukawa matrices and CKM mixing angles is guaranteed by the different powers of the flavon field. Using a triangular form for N and taking into account that the hierarchy in the left handed angles implies that $q_j > q_i$, we have that $N_{i<j} = (\theta^*/M)^{(q_j - q_i)} = \epsilon^{(q_j - q_i)}$. This form is forced from the requirement of invariance under the abelian symmetry of the canonically normalised Yukawa element. In the case of non-abelian symmetries the hierarchy in the Yukawa matrices is obtained from the smallness of vevs of the different flavon fields. In principle, only the vev defining the third generation can be $\mathcal{O}(1)$ while vevs in the direction of the second or first generation are $\leq \epsilon$. Given that the flavour structure of the Kähler matrices is necessarily generated in terms of the same flavon vevs, we have that any off-diagonal term in the Kähler matrices involves at least one power of the small vevs and hence $N_{i<j} \leq \epsilon$. A similar argument applies to the up quark RH sector, $N^{R,u}_{ii} \simeq 1$ and $N^{R,u}_{ik} \leq \varepsilon$ but in the down quark RH sector the expansion parameter must be the larger one, $\bar{\varepsilon}$, so $N^{R,d}_{ii} \simeq 1$ and $N^{R,d}_{ik} \leq \bar{\varepsilon}$.

In fact it is easy to prove that for the hierarchical textures of interest here the leading correction to a given Yukawa element is suppressed by at least $\mathcal{O}(\epsilon^2)$.[3] With the underlying family symmetry ordering the correction we know that, before symmetry breaking, the operator giving rise to the correction to a given element must transform in the same way under the family symmetry as the leading term. We have just proved that the difference of the Kähler transformations from the identity is at least of $O(\epsilon)$. Furthermore corrections to Y_{ij} after transformations to canonical Kähler with upper triangular matrices come only from Y_{kj} with $i > k$ and $Y_{kj} < Y_{ij}$. This implies that a new contribution to Y^t_{ij} is subdominant relative to Y_{ij} at least by $O(\epsilon)$ where $\epsilon =< \theta > /M$. As θ transforms non-trivially under the family symmetry, to maintain the symmetry property of the leading term, this relative correction must be given by a combination of fields which transforms as a singlet, that is at least of the form $\theta\theta^\dagger$ and hence of $\mathcal{O}(\epsilon^2)$. This result applies to hierarchical Yukawa structures. For the case that the $(2,3)$ element saturates the bound of eq. (3.1) it violates the condition of hierarchical Yukawa couplings and our conclusions above do not apply. In what follows we consider this possibility separately.

[2]Here we have implicitly assumed that $\epsilon = \langle\theta\rangle/M$ is the fundamental expansion parameter. If this is not true and the true expansion parameter is larger (e.g. θ is itself generated by a higher dimension term $\phi.\phi/M$) one should allow for the possibility that the expansion parameter in the *Kähler* sector is the larger one (e.g. $< \phi > /M$).

[3]This was first shown in the particular case of abelian flavour symmetries in ref. [18]

Using this we will now calculate the canonical Yukawa through eq. (3.2). Although we have started with the superpotential generating the form of eq. (3.1) in the symmetry basis we have the freedom to use any basis when calculating the effects on physical quantities. It is convenient to go to the Cholesky form when determining the effects of the Kähler potential and we use an upper triangular form for the Kähler rescaling matrix in the LH sector with $N^L_{ki} = 0$ for $i < k$. The corrections to a given element of the Yukawa matrix induced by the transformation to canonical Kähler are given by $N^*_{ki} Y_{kj}$.

3.2.1 No additional texture zeros

We first consider the case without additional texture zeros so that all of a, b, c, d are of $\mathcal{O}(1)$. Taking into account that $Y_{kj} < Y_{ij}$ for $i > k$ and $j \geq i$ we conclude that $N_{ki} Y_{kj} < Y_{ij}$. Therefore, these corrections are always sub-dominant in ϵ. This is not yet sufficient to prove that the transformation to the canonical left handed Kähler basis does not change the observable mixings and masses because they could be sensitive to elements of Y below the diagonal. Given the bounds of eq. (3.1) the only terms that could be modified by Kähler corrections are the $(2,1)$, $(3,1)$ and $(3,2)$ terms. For instance, for $Y_{3,1} < Y_{2,1}$ the Kähler correction can dominate the $(3,1)$ element. However in this case, from the structure in eq. (3.1) and with $N_{i<j} \leq \epsilon$, $Y^t_{3,1} \leq \epsilon^4$. Clearly this is too small to affect masses or LH mixing angles at leading order. It can be easily checked that the same is true in the case of the $(2,1)$ and $(3,2)$ elements. As we have discussed, for the hierarchical textures of interest here, the leading correction to a given Yukawa element is suppressed by at least $\mathcal{O}(\epsilon^2)$.

One might worry that the condition $Y_{kj} < Y_{ij}$ for $i > k$ and $j \geq i$ is too strong and that what are constrained are the elements after Kähler mixing, i.e. $Y^t_{kj} < Y^t_{ij}$ for $i > k$ and $j \geq i$ and the condition on Y_{kj} is not satisfied. However this is inconsistent. To see this note that the phenomenological structure of Y^t_{kj} in eq. (3.1) would correspond both to the basis of canonical Kähler with upper triangular transformations or to the basis of "standard" canonical transformations. This is due to the fact that both basis are related by a small rotation which does not change the order of the elements if the departure of the original Kähler metric from the identity is also hierarchical as expected in models with a spontaneously broken family symmetry. Thus we still have $N_{ik} \leq \epsilon$ for $i \neq k$. Therefore, we would need $Y_{kj} > Y_{ij}$ for $i > k$, or more exactly the power in ϵ of Y_{kj} is smaller than the power in ϵ of Y_{ij} for $i > k$ so that $N^*_{ki} Y_{kj} > Y_{ij}$ is possible. However in this case we necessarily have $Y^t_{kj} = Y_{kj} > Y_{ij} + N_{ik} Y_{kj} = Y^t_{ij}$ for $i > k$ and $j \geq i$ (neglecting smaller contributions from Y_{mj} with $m < k$ if present) and we arrive to an inconsistency with the initial statement $Y^t_{kj} < Y^t_{ij}$. Thus even with the weaker condition we need $Y_{kj} < Y_{ij}$ for $i > k$ and $j \geq i$.

So far we have discussed the transformations to canonical Kähler for the left handed fields. Now, we have to proceed exactly in the same way for the right-handed transformation. Clearly, if the Yukawa structures are also hierarchical we can perform the same analysis using upper triangular matrices and we would again arrive to the conclusion that corrections from the Kähler to any Yukawa element are always sub-dominant at least by ϵ^2 ($\epsilon = \bar{\varepsilon}, \varepsilon$ for $Y = Y_d, Y_u$). There is an exception to this conclusion if Y_{23} does not preserve the hierarchical structure and is of $\mathcal{O}(1)$ saturating the bound in eq. (3.1). In this case it is possible that $N^R_{23} = \mathcal{O}(1)$ and therefore corrections $\mathcal{O}(1)$ to Y_{i3} are still possible. Even in this case, it is clear that we can never modify the order in ϵ of the different elements of the Yukawa matrix, all it can do is to change the $\mathcal{O}(1)$ coefficients of the Y_{i3} elements. To determine whether this special case is possible one needs to know Y_{32} and this can be done through measurement of flavour changing neutral currents [19, 20] or lepton flavour violation [21].

Thus, using the triangular form, we have shown that the Kähler corrections to the Yukawa matrix are sub-dominant for hierarchical Yukawa matrices. In the next section we prove that this is also true for the observable mixing angles and mass eigenstates.

3.2.2 Additional texture zeros

A special situation occurs when one of a, b, c, d is $< \mathcal{O}(1)$ giving rise to an approximate texture zero. This can spoil the hierarchical structure of our Yukawa textures, $Y_{kj} < Y_{ij}$ for $i > k$ and $j \geq i$ and therefore must be analysed separately. An example of the origin of such zeros occurs in spontaneously broken abelian theories through the so-called holomorphic zeros [5]. In this case the symmetry breaking is through flavon field(s) carrying only one sign of charge (say negative) and then a net negative charge of the fermionic fields cannot be compensated with insertions of the flavon field because, due to the holomorphicity of the superpotential, the charged conjugated flavon can not be used. However the Kähler potential is non holomorphic and therefore these zeros can be filled after the transformation to the canonical basis.

As before, if we are only interested in the physical effects of this texture zero filling we can choose a convenient basis [5]. Once more our choice of upper triangular matrices is especially simple. In a hierarchical texture we can have a texture zero in any position of the matrix except in Y_{33} which is necessarily $\mathcal{O}(1)$. Although it is clear that the texture zeros can be filled in by the Kähler corrections we can immediately use the analysis presented above to show that physical measureables will not be affected by these corrections. The point, as is explicitly demonstrated in the next section, is that the form of eq. (3.1) gives the value of each entry of the Yukawa matrix that has a leading effect on a mass or a mixing angle. If the entry is larger than the value shown it will give a mass or mixing angle in conflict with the measured value. If the entry is smaller it will only contribute to measureable quantities at subleading order.

In the previous section we showed that, for the case of hierarchical textures, the Kähler corrections only contribute to the Yukawa matrix elements suppressed relative to the order shown in eq. (3.1) by at least $\mathcal{O}(\epsilon^2)$. For example we can see that a zero in Y_{11} is never filled by any other element. In the same way a zero in Y_{12} or Y_{21} is only filled by a non-zero entry in Y_{11}. Taking into account the constraints from the determinant of the Yukawa matrix, $Y_{11} \leq \epsilon^4$ and in the hierarchical case with $N^{L(R)}_{i\neq j} \leq \epsilon$ this implies that they can only be filled at $\mathcal{O}(\epsilon^5)$. In the same way Y_{13} Y_{31} and Y_{22} can only be filled at $\mathcal{O}(\epsilon^4)$ $(Y_{12}, Y_{21} \leq \epsilon^3)$. Finally a zero in Y_{23} or Y_{32} implies that $Y_{22} = \epsilon^2$ and hence these zeros can be filled at most at $\mathcal{O}(\epsilon^3)$. As we will now show, these subleading terms only contribute to physical quantities at subleading order even though the texture zero may be filled in. The only exception to this is when the hierarchical structure is spoilt through an $\mathcal{O}(1)$ term in Y_{23}. In this case, following the discussion given above, the Kähler corrections can contribute at $\mathcal{O}(1)$ to physical quantities.

4. Kähler corrections to the mass matrix eigenvalues and mixing angles

To complete our proof we need to demonstrate that the entries of eq. (3.1) are the smallest that can affect masses and mixing angles and thus the Kähler corrections, which we have shown are smaller than those of eq. (3.1), are necessarily subdominant in determining physical quantities.

4.1 Quark and charged lepton masses and mixing angles.

Since the Kähler corrections are wave function corrections which cannot change the rank of the mass matrix we know that they lead to multiplicative normalisations of the masses. For hierarchical Yukawa matrices the wave function normalisation has the form $N_{ik} = \delta_{ik} + \mathcal{O}(\leq \varepsilon)$ and this means the Kähler corrections to masses are necessarily sub-dominant. To see this explicitly, consider only the left handed canonical normalisation N_{ik} with N upper triangular. Now using eq. (3.2) the canonical Yukawa and the fact that the Yukawa and Kähler matrices are hierarchical in the left handed sector, the determinant of Y^t is,

$$\mathrm{Det}(Y^t) = \mathrm{Det}(N)\mathrm{Det}(\hat{Y}) \simeq (1 + \mathcal{O}(\leq \epsilon))\mathrm{Det}(\hat{Y})\,.$$

Moreover, from the hierarchical structure in eq. (3.1) we know that any element of the matrix is corrected only at $\mathcal{O}(\leq \epsilon^2)$ under the transformations to canonical left-handed Kähler. In particular, the heaviest eigenvalue in Y^t will be still be $1 + \mathcal{O}(\leq \epsilon^2)$. Therefore this implies that the product of the two lightest eigenvalues can only be changed at $\mathcal{O}(\leq \epsilon^2)$. Finally the second eigenvalue is basically obtained from the lightest eigenvalue of the $(2,3)$ submatrix and thus we obtain again that any change to this eigenvalue will be sub-dominant in ϵ and therefore the same is true for the first generation eigenvalue.

In the case of a non-hierachical structure in the $(2,3)$ entry with N^R_{23} of $\mathcal{O}(1)$ we expect $\mathrm{Det}(N^R)$ to be $\mathcal{O}(1)$ barring accidental cancellations. In this case the corrections to the eigenvalues, while still not changing their order in ϵ, could be $\mathcal{O}(1)$.

Concerning the mixing angles, with the use of triangular matrices we have not changed the hierarchical structure of the Yukawa matrices. Hence, we can still use the usual perturbative expansion. In this way, after the transformations to left handed canonical Kähler we have,

$$\begin{aligned}\theta_{23} &= \theta^d_{23} - \theta^u_{23} = \frac{(Y^d_{23})^t}{(Y^d_{33})^t} - \frac{(Y^u_{23})^t}{(Y^u_{33})^t} = \frac{\hat{Y}^d_{23}(1+\mathcal{O}(\varepsilon^2))}{\hat{Y}^d_{33}(1+\mathcal{O}(\varepsilon^2))} - \frac{\hat{Y}^u_{23}(1+\mathcal{O}(\varepsilon^2))}{\hat{Y}^u_{33}(1+\mathcal{O}(\varepsilon^2))} \\ &= \hat{\theta}_{23}(1+\mathcal{O}(\varepsilon^2)) \end{aligned} \tag{4.1}$$

the discussion is identical for the θ_{13} mixing angle. The case of θ_{12} is slightly more complicated, now we have,

$$\theta^d_{12} = \frac{(Y^d_{12})^t}{(Y^d_{22})^t - (Y^d_{23})^t(Y^d_{32})^t} \tag{4.2}$$

where the denominator is really the Y^d_{22} element in the basis where we have already diagonalised the 2, 3 sector, and it is approximately equal to $m_s/m_b = \bar{\varepsilon}^2$. However, we know that both $(Y^d_{22})^t \leq \bar{\varepsilon}^2(1 + \mathcal{O}(\varepsilon^2))$ and $(Y^d_{23})^t(Y^d_{32})^t \leq \bar{\varepsilon}^2(1 + \mathcal{O}(\varepsilon^2))$. This means that the denominator can also be corrected only at $\mathcal{O}(\varepsilon^2)$, then we have,

$$\theta^d_{12} = \frac{\hat{Y}^d_{12}(1 + \mathcal{O}(\varepsilon^2))}{(\hat{Y}^d_{22} - \hat{Y}^d_{23}\hat{Y}^d_{32})(1 + \mathcal{O}(\varepsilon^2))} = \hat{\theta}^d_{12}(1 + \mathcal{O}(\varepsilon^2)) \tag{4.3}$$

doing the same for θ^u_{12} we arrive immediately to $\theta_{12} = \theta^d_{12} - \theta^u_{12} = \hat{\theta}_{12}(1 + \mathcal{O}(\varepsilon^2))$.

Moreover, it is easy to check that the effect of the transformation to canonical Kähler for the right handed fields on the left handed mixings is usually negligible. To see this, we consider the limit of trivial left handed Kähler and nontrivial right-handed Kähler. Then, we consider the diagonalisation of the hermitean matrix H^t,

$$H^t = Y^t(Y^t)^\dagger = \hat{Y}N_R N_R^\dagger \hat{Y}^\dagger = \hat{Y}K^{-1}\hat{Y}^\dagger = \hat{V}_L^\dagger \hat{M}_f \hat{V}_R K^{-1}\hat{V}_R^\dagger \hat{M}_f V_L \equiv \hat{V}_L^\dagger \hat{M}_f \tilde{K}^{-1}\hat{M}_f \hat{V}_L \,,$$

where we have written $\hat{Y} = \hat{V}_L^\dagger \hat{M}_f \hat{V}_R$ and reabsorbed the right-handed rotation in $\tilde{K}^{-1}$, i.e. we have written the inverse of the Kähler in the basis of right handed mass eigenstates. Now it is trivial to see that the matrix diagonalising H^t will be the product of V_L with the matrix diagonalising $\hat{M}_f\tilde{K}^{-1}\hat{M}_f$. As we have seen $\hat{M}_f$ are approximately equal to the eigenvalues of the total Yukawa matrix, this implies that $\hat{M}_f\tilde{K}^{-1}\hat{M}_f$ is strongly hierarchical and then the mixing angles diagonalising this matrix will be,

$$\tilde{\theta}_{i3} \simeq \frac{m_i m_3(\tilde{K}^{-1})_{i3}}{m_3^2(\tilde{K}^{-1})_{33}} \qquad \tilde{\theta}_{12} \simeq \frac{m_1 m_2(\tilde{K}^{-1})_{12}}{m_2^2\left((\tilde{K}^{-1})_{22} - \dfrac{|(\tilde{K}^{-1})_{23}|^2}{(\tilde{K}^{-1})_{33}}\right)}$$

therefore these contributions are suppressed both by the smallness of off-diagonal entries in the Kähler with respect to diagonal ones and by ratios of fermion masses. This last suppression is usually enough to make $\tilde{\theta}_{ij} \ll \theta_{ij}$ and then we can safely neglect the effect of right handed transformation in left handed mixings.

The exception to this rule arises when the right handed Kähler in the basis of right handed mass eigenstates is not hierarchical and has $\mathcal{O}(1)$ entries in K_{23}, K_{22} and K_{33}. In this case the correction to the angle θ_{23} from the down quark right handed Kähler could be of leading order as both θ_{23} and m_s/m_b are $\mathcal{O}(\bar{\varepsilon}^2)$. Still this situation can be understood as an exception to the main rule we formulated above. The correction from the right handed Kähler in the left handed mixing angles would still be of the same order as the contribution from the non-canonical Yukawa matrix and therefore would only modify the unknown $\mathcal{O}(1)$ coefficients. Usually, we find this structure in U(1) models with lopsided Yukawa textures [22]. These models depend precisely on the existence of different $\mathcal{O}(1)$ coefficients in the elements of the Yukawa texture to obtain the correct masses and mixing angles. However, the U(1) symmetry has no control on these $\mathcal{O}(1)$ coefficients and so this means that we do not need to worry about these effects. Only in a theory where we can control these unknown coefficients we should worry about the effects of this right-handed field redefinition.

4.2 Neutrino masses and mixing angles

The case of neutrino masses can be analysed with similar techniques. In this case, we obtain the effective Majorana mass matrix for the left handed neutrinos through the seesaw mechanism. The neutrino mass matrix structure has the form

$$L_\nu = -\nu_{L\,i} Y^\nu_{ij} \nu^c_{R\,j} - \frac{1}{2}\nu_{R\,i} M_{R\,ij} \nu^c_{R\,j} + \text{h.c.}$$

giving the effective Majorana mass matrix of the effective low energy neutrinos, M_ν of the form

$$M_\nu = \chi_\nu \ (v \sin\beta)^2 = Y^\nu (M_R)^{-1} Y^{\nu\,T} \ \ (v\sin\beta)^2 \,.$$

The transformation properties of the effective neutrino mass matrix under the transformations to canonical Kähler for both left handed and right handed fields is given by

$$\begin{aligned} \chi^t_\nu &= Y^{\nu\,t} (M^t_R)^{-1} (Y^{\nu\,t})^T = N^T_L \ \hat{Y}^\nu \ N_R (N_R)^{-1} \ M^{-1}_R \ (N^T_R)^{-1} N^T_R \ \hat{Y}^{\nu\,T} \ N_L \\ &= N^T_L \ \hat{Y}^\nu \ \hat{M}^{-1}_R \ \hat{Y}^{\nu\,T} \ N_L = N^T_L \ \hat{\chi}_\nu \ N_L \,. \end{aligned} \tag{4.4}$$

Hence, we see that the effective neutrino coupling χ_ν is transformed only by the left handed canonical transformations and the right-handed transformations cancel exactly.

However the neutrino sector can be special because in this case, we do not know much about the hierarchy of the leptonic Yukawa couplings Y^ν and Y^e. In fact we can find two different situations:

1. Y^ν and Y^e are hierarchical and $Y_{kj} < Y_{ij}$ for $i > k$ and $j \geq i$. This is this situation in realistic non-abelian flavour theories explored to date [14].

2. Y^ν or Y^e have two rows of similar size. We can find this situation in some U(1) models [23].

In case 1 the Kähler metric is also very close to the identity with small off-diagonal entries. Therefore we can choose N_L to be upper triangular with $(N_L)_{ii} \simeq 1$ and $(N_L)_{ij} \leq \epsilon$. Then both Y_ν and Y_e are only changed at higher order in ϵ and neutrino masses and mixings are only changed at sub-dominant order. In the case of non-abelian symmetries χ^t_ν and Y^e are changed at most at order ε^2. Then we can immediately use the standard formulae for the neutrino mixings compiled in Ref. [24]. For all the different cases compatible with hierarchical rows in the lepton Yukawa matrix, we can immediately see that neutrino mixings will only be changed at sub-leading order. Although small, this might still be relevant for the difference of the solar mixing angle from maximality [25].

Case 2 arises if two left handed fields have identical flavour symmetry charges. As a result the Kähler metric will have large mixing between these two fields and therefore $\mathcal{O}(1)$ off-diagonal entries. In this case, it is possible to modify the $\mathcal{O}(1)$ coefficients in the different elements of the canonical Yukawa matrices, but the order in ϵ of these entries is not changed. Therefore, in this case, it is possible to generate changes at leading order in neutrino masses and mixings. This corresponds again to the case where right-handed mixing angles can modify left-handed mixings in the quark sector. Since only the $\mathcal{O}(1)$ coefficients are modified these corrections do not change the predicted structure if the family symmetry does not predict the value of these coefficients.

JHEP07(2005)049

4.3 Soft SUSY breaking masses and mixing angles

Finally, we would also like to comment on the effects of the Kähler transformations on the soft breaking masses which may give rise to dangerous flavour changing neutral current processes [19]. Notice that the F-term contributions to soft breaking masses in supergravity are closely related to the Kähler potential [26]. In fact the non canonical soft breaking masses are,

$$\hat{m}^2_{\overline{a}b} = m^2_{3/2} K_{\overline{a}b} - F_{\overline{m}} \left(\partial_{\overline{m}} \partial_n K_{\overline{a}b} - \partial_{\overline{m}} K_{\overline{a}c} (K^{-1})_{c\overline{d}} \partial_n K_{\overline{d}b} \right) F_n \,.$$

To obtain the canonical soft breaking masses we have to multiply this matrix by the inverse of the square root of K, $m^2 = (K^{-1/2})^\dagger \hat{m}^2 K^{-1/2}$. Then we obtain,

$$\begin{aligned} m^2 &= m^2_{3/2} \mathbf{1} - (K^{-1/2})^\dagger F_{\overline{m}} \left(\partial_{\overline{m}} \partial_n K - \partial_{\overline{m}} K (K^{-1}) \partial_n K \right) F_n K^{-1/2} \\ &\equiv m^2_{3/2} \mathbf{1} - N^\dagger F_{\overline{m}} \left(\partial_{\overline{m}} \partial_n K - \partial_{\overline{m}} K (K^{-1}) \partial_n K \right) F_n N \,. \end{aligned}$$

Therefore we see that we have a universal contribution proportional to $m^2_{3/2}$ plus other terms which in principle will depend on flavour. These terms depend on the derivatives of the Kähler potential with respect to fields with non vanishing F-terms.

If the field with non-vanishing F-term is a hidden sector field it must be neutral under the flavour symmetry and therefore the structure in powers of ϵ of $\partial_{\overline{m}} \partial_n K$ or $\partial_{\overline{m}} K$ will be the same as the structure of K. However, factors $\mathcal{O}(1)$ can be different and indeed can sometimes be zero. The important point is that no terms larger in powers of ϵ are generated than are in K itself. Due to this difference in the $\mathcal{O}(1)$ coefficients the product $(K^{-1/2})^\dagger \partial_{\overline{m}} K K^{-1/2}$ will be different from the identity, but will be bounded by the same power in ϵ as the original K matrix [8].

Another possibility is that the field with non-vanishing F-term is a flavon field with non-trivial quantum numbers under the flavour symmetry. As shown in [27], the natural size for F_θ for θ a flavon field is $m_{3/2} \langle \theta \rangle$, although it can be smaller depending on the characteristics of the scalar potential. In this case, we also have that $F_{\overline{m}} \partial_{\overline{m}} K$ cannot generate terms larger in powers of ϵ than the terms initially present in K itself and the conclusion above still applies.

We have also to consider the possibility of a non-vanishing flavour D-term contributing to the soft masses. Although this possibility is extremely dangerous for the phenomenology of flavour changing neutral currents (FCNCs) it can be realised for heavy sfermion masses in some abelian flavour models. In this case we obtain a new contribution to the soft masses,

$$(\hat{m}^2_{\overline{a}b})^D = g\, q_b\, K_{\overline{a}b}\, \langle D \rangle$$

with q_b the charge of the field ϕ_b under the $\mathrm{U}(1)_{fl}$ symmetry. Notice that due to the dependence on the charges of the different fields this contribution to the soft masses is not diagonalised when we make the transformation to the basis of canonical Kähler and therefore it gives rise to new FCNC effects.

To analyse these FCNC effects it is convenient to work in the SCKM basis where the corresponding Yukawa matrix is diagonal. Therefore, to obtain the sfermion mass matrix in the SCKM basis we have to do two trasformations. First we go to the basis of canonical Kähler with our triangular matrices and second we diagonalise the corresponding Yukawa matrix with a rotation of the full superfield. Now, we can compare the effects of the transformations to the basis of canonical Kähler with the effects of the second transformation to the SCKM basis. First it is easy to see that in U(1) models the structure in ϵ of our triangular Kähler transformations are always smaller or equal that the corresponding rotation diagonalising the Yukawa matrix. For instance, the left handed Kähler transformation is usually of the same order as the left handed rotation diagonalising the up quark Yukawa matrix and smaller than the left handed rotation diagonalising the down quark Yukawa. If the diagonal elements of the Kähler metric are $\mathcal{O}(1)$, this means that the corrections to offdiagonal elements that we obtain from the transformations to the SCKM basis are larger or equal than the corrections obtained in the transformation to the canonical basis. As before, if we are not interested in coefficients $\mathcal{O}(1)$, we can also ignore the effects of transformation to canonical Kähler in the soft breaking masses.

5. Conclusions

In this letter we have studied the effects of the transformations to the canonical Kähler basis on the Yukawa textures for quarks and leptons and their contributions to physical masses and mixing angles. We have developed a simple formalism that allows a straightforward calculation of the necessary Kähler transformations and simplifies enormously the phenomenological analysis. Using this formalism we have proved that, in the case of models with a hierarchical structure of the Yukawa matrices, the corrections obtained through the transformations to canonical Kähler are always suppressed by a factor $\leq \epsilon^2$ with ϵ the expansion parameter in the Yukawa matrix. This implies that, in this case, fermionic masses and mixing angles receive only corrections at ϵ^2 from the Kähler transformations. We have seen that although texture zeros can be filled by transformations to canonical Kähler the physical effects of this texture zero filling are only subdominant corrections in ϵ to observable masses and mixing angles. We have also discussed some exceptions to the case of completely hierarchical Yukawa matrices where some corrections at leading order are possible. In any case, we have seen that in these models only unknown $\mathcal{O}(1)$ coefficients are modified. We have also shown that the corrections to the scalar soft breaking mass matrices can only change the unknown $\mathcal{O}(1)$ coefficients. We conclude that in the large class of models considered here the leading order superpotential couplings in the noncanonical Kähler basis are essentially unchanged when transformed to the canonical Kähler basis.

Acknowledgments

L.V.S. would like to thank G. Kane for helpful discussions. O.V. acknowledges partial support from the spanish MCYT FPA2002-00612 and Andrea Romanino for useful discusions.

Note added. During completion of this work, we learnt about two groups [28, 29] working along the same lines with different techniques. Our conclusions agree in the points where the analysis overlap. In ref. [28], the authors provide an exact formula relating the "naive" CKM and MNS matrices to the physical matrices. They show that the effects of canonical normalisation are subdominant in the case of hierarchical matrices in agreement with the present analysis which uses somewhat simpler mathematical techniques to obtain the transformation to the canonical Kähler basis. In addition we analyse the effects of canonical normalisation in the sfermion mass matrices and we find that these transformations do not change the structure of the sfermion mass matrices.

References

[1] C.D. Froggatt and H.B. Nielsen, *Hierarchy of quark masses, Cabibbo angles and CP-violation*, *Nucl. Phys.* **B 147** (1979) 277.

[2] For a review and further references see: G.G. Ross, *Models of fermion masses*, published in *TASI 2000* J.L. Rosner ed., World Scientific, New Jersey, 2001.

[3] G. Feinberg, P. Kabir and S. Weinberg, *Transformation of muons into electrons*, *Phys. Rev. Lett.* **3** (1959) 527.

[4] M. Leurer, Y. Nir and N. Seiberg, *Mass matrix models: the sequel*, *Nucl. Phys.* **B 420** (1994) 468 [hep-ph/9310320].

[5] E. Dudas, S. Pokorski and C.A. Savoy, *Yukawa matrices from a spontaneously broken abelian symmetry*, *Phys. Lett.* **B 356** (1995) 45 [hep-ph/9504292]; *Soft scalar masses in supergravity with horizontal* $U(1)_X$ *gauge symmetry*, *Phys. Lett.* **B 369** (1996) 255 [hep-ph/9509410].

[6] H.K. Dreiner and M. Thormeier, *Supersymmetric Froggatt-Nielsen models with baryon- and lepton-number violation*, *Phys. Rev.* **D 69** (2004) 053002 [hep-ph/0305270].

[7] S.F. King and I.N.R. Peddie, *Canonical normalisation and Yukawa matrices*, *Phys. Lett.* **B 586** (2004) 83 [hep-ph/0312237].

[8] I. Jack, D.R.T. Jones and R. Wild, *Yukawa textures and the* μ*-term*, *Phys. Lett.* **B 580** (2004) 72 [hep-ph/0309165].

[9] M. Dine, R.G. Leigh and A. Kagan, *Flavor symmetries and the problem of squark degeneracy*, *Phys. Rev.* **D 48** (1993) 4269 [hep-ph/9304299];
A. Pomarol and D. Tommasini, *Horizontal symmetries for the supersymmetric flavor problem*, *Nucl. Phys.* **B 466** (1996) 3 [hep-ph/9507462];
L.J. Hall and H. Murayama, *A geometry of the generations*, *Phys. Rev. Lett.* **75** (1995) 3985 [hep-ph/9508296];
R. Barbieri, G.R. Dvali and L.J. Hall, *Predictions from a* U(2) *flavour symmetry in supersymmetric theories*, *Phys. Lett.* **B 377** (1996) 76 [hep-ph/9512388];
Z. Berezhiani, *Unified picture of the particle and sparticle masses in SUSY GUT*, *Phys. Lett.* **B 417** (1998) 287 [hep-ph/9609342];
E. Dudas, C. Grojean, S. Pokorski and C.A. Savoy, *Abelian flavour symmetries in supersymmetric models*, *Nucl. Phys.* **B 481** (1996) 85 [hep-ph/9606383];
R. Barbieri, L.J. Hall, S. Raby and A. Romanino, *Unified theories with* U(2) *flavor symmetry*, *Nucl. Phys.* **B 493** (1997) 3 [hep-ph/9610449].

JHEP07(2005)049

[10] Y. Nir and N. Seiberg, *Should squarks be degenerate?*, *Phys. Lett.* **B 309** (1993) 337 [hep-ph/9304307];
Y. Nir and G. Raz, *Quark squark alignment revisited*, *Phys. Rev.* **D 66** (2002) 035007 [hep-ph/0206064].

[11] Weak basis transformations in the context of CP violation are used for instance in:
C. Jarlskog, *Commutator of the quark mass matrices in the standard electroweak model and a measure of maximal CP-violation*, *Phys. Rev. Lett.* **55** (1985) 1039;
J. Bernabeu, G.C. Branco and M. Gronau, *CP restrictions on quark mass matrices*, *Phys. Lett.* **B 169** (1986) 243;
G.C. Branco and V.A. Kostelecky, *CP-violation in supergravity models*, *Phys. Rev.* **D 39** (1989) 2075;
F.J. Botella and J.P. Silva, *Jarlskog-like invariants for theories with scalars and fermions*, *Phys. Rev.* **D 51** (1995) 3870 [hep-ph/9411288];
O. Lebedev, *CP-violating invariants in supersymmetry*, *Phys. Rev.* **D 67** (2003) 015013 [hep-ph/0209023];
S. Davidson and R. Kitano, *Leptogenesis and a jarlskog invariant*, *JHEP* **03** (2004) 020 [hep-ph/0312007].

[12] F.J. Botella, M. Nebot and O. Vives, *Invariant approach to flavour-dependent CP-violating phases in the MSSM*, hep-ph/0407349.

[13] R.G. Roberts, A. Romanino, G.G. Ross and L. Velasco-Sevilla, *Precision test of a fermion mass texture*, *Nucl. Phys.* **B 615** (2001) 358 [hep-ph/0104088].

[14] S.F. King and G.G. Ross, *Fermion masses and mixing angles from* SU(3) *family symmetry*, *Phys. Lett.* **B 520** (2001) 243 [hep-ph/0108112]; *Fermion masses and mixing angles from* SU(3) *family symmetry and unification*, *Phys. Lett.* **B 574** (2003) 239 [hep-ph/0307190].

[15] G.G. Ross, L. Velasco-Sevilla and O. Vives, *Spontaneous CP-violation and non-abelian family symmetry in SUSY*, *Nucl. Phys.* **B 692** (2004) 50 [hep-ph/0401064].

[16] L.E. Ibáñez and G.G. Ross, *Fermion masses and mixing angles from gauge symmetries*, *Phys. Lett.* **B 332** (1994) 100 [hep-ph/9403338].

[17] See for example: P. Binetruy, S. Lavignac and P. Ramond, *Yukawa textures with an anomalous horizontal abelian symmetry*, *Nucl. Phys.* **B 477** (1996) 353 [hep-ph/9601243].

[18] G. Eyal and Y. Nir, *Approximate CP in supersymmetric models*, *Nucl. Phys.* **B 528** (1998) 21 [hep-ph/9801411].

[19] F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, *A complete analysis of fcnc and CP constraints in general SUSY extensions of the standard model*, *Nucl. Phys.* **B 477** (1996) 321 [hep-ph/9604387];
A. Masiero and O. Vives, *New physics in CP-violation experiments*, *Ann. Rev. Nucl. Part. Sci.* **51** (2001) 161 [hep-ph/0104027].

[20] T. Moroi, *CP-violation in* $B/D \to \phi K(s)$ *in SUSY GUT with right-handed neutrinos*, *Phys. Lett.* **B 493** (2000) 366 [hep-ph/0007328];
N. Akama, Y. Kiyo, S. Komine and T. Moroi, *CP-violation in kaon system in supersymmetric* SU(5) *model with seesaw-induced neutrino masses*, *Phys. Rev.* **D 64** (2001) 095012 [hep-ph/0104263];
D. Chang, A. Masiero and H. Murayama, *Neutrino mixing and large CP-violation in B physics*, *Phys. Rev.* **D 67** (2003) 075013 [hep-ph/0205111];

M. Ciuchini, E. Franco, A. Masiero and L. Silvestrini, *$B \to S$ transitions: a new frontier for indirect SUSY searches*, *Phys. Rev.* **D 67** (2003) 075016 [hep-ph/0212397];
R. Harnik, D.T. Larson, H. Murayama and A. Pierce, *Atmospheric neutrinos can make beauty strange*, *Phys. Rev.* **D 69** (2004) 094024 [hep-ph/0212180];
S. Khalil and E. Kou, *On supersymmetric contributions to the CP asymmetry of the $B \to \phi K(s)$*, *Phys. Rev.* **D 67** (2003) 055009 [hep-ph/0212023];
G.L. Kane et al., *$B/d \to \phi K(s)$ CP asymmetries as an important probe of supersymmetry*, *Phys. Rev. Lett.* **90** (2003) 141803 [hep-ph/0304239];
D. Chakraverty, E. Gabrielli, K. Huitu and S. Khalil, *Chargino contributions to the CP asymmetry in $B \to \phi K(s)$ decay*, *Phys. Rev.* **D 68** (2003) 095004 [hep-ph/0306076];
M. Ciuchini, A. Masiero, L. Silvestrini, S.K. Vempati and O. Vives, *Grand unification of quark and lepton FCNCs*, *Phys. Rev. Lett.* **92** (2004) 071801 [hep-ph/0307191].

[21] J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, *Lepton-flavor violation via right-handed neutrino Yukawa couplings in supersymmetric standard model*, *Phys. Rev.* **D 53** (1996) 2442 [hep-ph/9510309];
J. Hisano and D. Nomura, *Solar and atmospheric neutrino oscillations and lepton flavor violation in supersymmetric models with the right-handed neutrinos*, *Phys. Rev.* **D 59** (1999) 116005 [hep-ph/9810479];
I. Masina and C.A. Savoy, *Sleptonarium (constraints on the CP and flavour pattern of scalar lepton masses)*, *Nucl. Phys.* **B 661** (2003) 365 [hep-ph/0211283];
S. Baek, T. Goto, Y. Okada and K.I. Okumura, *Neutrino oscillation, SUSY GUT and b decay*, *Phys. Rev.* **D 63** (2001) 051701 [hep-ph/0002141];
S. Baek, T. Goto, Y. Okada and K.-i. Okumura, *Muon anomalous magnetic moment, lepton flavor violation and flavor changing neutral current processes in SUSY GUT with right-handed neutrino*, *Phys. Rev.* **D 64** (2001) 095001 [hep-ph/0104146];
T. Blazek and S.F. King, *Muon anomalous magnetic moment and $\tau \to \mu\gamma$ in a realistic string-inspired model of neutrino masses*, *Phys. Lett.* **B 518** (2001) 109 [hep-ph/0105005];
A. Masiero, S.K. Vempati and O. Vives, *Seesaw and lepton flavour violation in SUSY* SO(10), *Nucl. Phys.* **B 649** (2003) 189 [hep-ph/0209303];
J. Hisano and Y. Shimizu, *GUT relation in neutrino induced flavor physics in SUSY* SU(5) *GUT*, *Phys. Lett.* **B 565** (2003) 183 [hep-ph/0303071];
C.-S. Huang, T.J. Li and W. Liao, *Neutrino bilarge mixing and flavor physics in the flipped* SU(5) *model*, *Nucl. Phys.* **B 673** (2003) 331 [hep-ph/0304130].

[22] G. Altarelli and F. Feruglio, *A simple grand unification view of neutrino mixing and fermion mass matrices*, *Phys. Lett.* **B 451** (1999) 388 [hep-ph/9812475].

[23] S.F. King, *Atmospheric and solar neutrinos from single right-handed neutrino dominance and* U(1) *family symmetry*, *Nucl. Phys.* **B 562** (1999) 57 [hep-ph/9904210]; *Large mixing angle msw and atmospheric neutrinos from single right-handed neutrino dominance and* U(1) *family symmetry*, *Nucl. Phys.* **B 576** (2000) 85 [hep-ph/9912492].

[24] S.F. King, *Leptogenesis-MNS link in unified models with natural neutrino mass hierarchy*, *Phys. Rev.* **D 67** (2003) 113010 [hep-ph/0211228]; *Constructing the large mixing angle mns matrix in see-saw models with right-handed neutrino dominance*, *JHEP* **09** (2002) 011 [hep-ph/0204360].

[25] Y. Nir, *Pseudo-Dirac solar neutrinos*, *JHEP* **06** (2000) 039 [hep-ph/0002168].

[26] S.K. Soni and H.A. Weldon, *Analysis of the supersymmetry breaking induced by $N = 1$ supergravity theories*, *Phys. Lett.* **B 126** (1983) 215.

[27] G.G. Ross and O. Vives, *Yukawa structure, flavour and CP-violation in supergravity*, *Phys. Rev.* **D 67** (2003) 095013 [hep-ph/0211279].

[28] J.R. Espinosa and A. Ibarra, *Flavour symmetries and Kähler operators*, *JHEP* **08** (2004) 010 [hep-ph/0405095].

[29] I. Jack and D.R.T. Jones, *Yukawa textures and an anomaly-free* U(1), work in progress.